Time frequency methods

- Wigner de Ville distributions
- Spectrogram – sliding window Fourier transform
- Wavelets
- Adaptive Approximations by Matching Pursuit

Multivariate autoregressive model (MVAR)

- MVAR and Granger causality
- Directed Transfer Function DTF
- Partial Directed Coherence PDC
- Short-time Directed Transfer Function SDTF
Wigner de Ville Transform

Fourier Transform of autocorrelation function

\[
W_f(t, \xi) = \int f\left(t + \frac{\tau}{2}\right) \overline{f\left(t - \frac{\tau}{2}\right)} e^{-i\xi \tau} d\tau
\]
Time frequency methods relying on the decomposition into basic functions:

Spectrogram - sinusoids

Wavelet transform - wavelets

Adaptive approximations by matching pursuit – broad repertoir of functions, usually functions from Gabor family including sinusoids
Spectrogram – windowed Fourier Transform

\[ g_I(t) = g(t-u)e^{i\xi t} \]

\[ Sf(u, \xi) = \int_{-\infty}^{+\infty} f(t)g(t-u)e^{-i\xi t} \, dt \]
Wavelet Transform

\[ g_1(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) \]

\[ Sf(u,s) = \int_{-\infty}^{+\infty} f(t) \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) dt = f \ast g_s(u) \]
Matching Pursuit

Dictionary of basic waveforms can be generated by scaling translating and modulating basic function \( g_I(t) \)

\[
g_I(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) e^{i\xi t}
\]

\( s>0 \) - scale, \( \xi \) - frequency modulation, \( u \) - translation.

Index \( I = (\xi, s, u) \) describes the set of parameters

The dictionaries of windowed Fourier transform and wavelet transform can be derived as subsets of this dictionary, defined by certain restrictions on the choice of parameters:

- In case of the windowed Fourier transform, the scale \( s \) is constant - equal to the window length - and the parameters \( \xi \) and \( u \) are uniformly sampled.

- In case of WT the frequency modulation is limited by the restriction on the frequency parameter \( \xi = \xi_0 / s, \xi_0 = \text{const.} \)
Matching Pursuit

Introduction of algorithm, 1993

First application to biological signal 1994, 1995:

Improvement of the original algorithm removing bias of the dyadic sampling of the time frequency space:
Matching Pursuit

In MP repertoire of function is very broad. Best time-frequency resolution is obtained when the functions $g_i(t)$ belong to Gabor functions family.

Finding an optimal approximation of signal by functions from such a large family is a NP-hard problem (computationally intractable). Therefore a suboptimal iterative procedure is applied. In the first step of the iterative procedure we choose the vector $g_{I0}$ which gives the largest product with the signal $f(t)$:

$$f = \langle f, g_{I0} \rangle g_{I0} + R^1 f$$

Then the residual vector $R^1$ obtained after approximating $f$ in the direction $g_{I0}$ is decomposed in a similar way. The iterative procedure is repeated on the following obtained residues:

$$R^n f = \langle R^n f, g_{I_n} \rangle g_{I_n} + R^{n+1} f$$

In this way the signal $f$ is decomposed into a sum of time-frequency waveforms, chosen to match optimally the signal’s residues:

$$f = \sum_{n=0}^{m} \langle R^n f, g_{I_n} \rangle g_{I_n} + R^{n+1} f$$
Statistical bias of representation was removed by introduction of stochastic dictionaries for sleep spindles and noise.

Construction of time-frequency distribution from waveforms

\[ W[f,f](t,\omega) = \sum_{n=0}^{\infty} \left| \langle R^n f, g_{1n} \rangle \right|^2 W[g_{1n},g_{1n}](t,\omega) + \sum_{n=0}^{\infty} \sum_{m=0,m\neq n}^{\infty} \left| \langle R^n f, g_{1n} \rangle \langle R^m f, g_{1m} \rangle \right| W[g_{1n},g_{1m}](t,\omega) \]
Time-Frequency distributions — comparison of methods

Spectrogram

Discrete Wavelet Transform

Choi-Williams Distribution

Spectrogram

Composition of the Test Signal

Matching Pursuit
Time-Frequency distributions — comparison of methods

In simulations γ functions of the form given below were used.

\[ \gamma(t) = \gamma_0 t^3 e^{-2\pi t^2} \sin(2\pi ft) \]

Time-frequency (t-f) energy density distributions for: A: Matching Pursuit (MP), B: windowed Fourier transform (STFT), C: Wigner-deVille distribution (WVD), D: continuous wavelet transform (WT).
Noise influence
Time-frequency distribution of EEG power during finger movement

EEG power distribution in time & frequency, electrode C1, right hand finger movement, subject Dd
T-F distribution of EEG power during epileptic seizure

Resonance modes – specific characteristics of inner ear.

OAE energy distribution in time-frequency for tonal stimulation of frequencies shown above the pictures. The same resonance modes are excited by different tonal stimuli.
MP provides parametric description of signal structures.
Statistical evaluation of structures observed in sleep EEG
Superimposed Spindles
Matching Pursuit and Inverse Solutions.

As the input to inverse solutions different quantities can be used e.g.:
value of electric field on the scalp in a given time moment, spectral integral, time integral representing given structure. This quantities represent cumulative activity which may have different origin - e.g. spectral integral in the 10 - 14 Hz may represent not only sleep spindles but also alpha activity.

By means of multichannel matching pursuit (MMP) particular structures can be extracted from EEG activity e.g.: sleep spindles, epileptic spikes.
Sleep spindles as input values to inverse solutions
Inverse solutions for epileptic activity
MP - examples of applications:


Adaptive Approximations by MP

• Representation in one framework rhythmic and transient components of the signal

• Parametric description of signal in terms of parameters with clear physiological meaning

• High resolution time-frequency representation

• Detection of microstructure of signals

• Identification of components close in frequency and/or in time

• Selective input to inverse problem solutions
Multivariate methods of signal analysis

- MVAR model
- DTF (assumption of ergodicity)
  - simulations
  - examples of applications
- SDTF (ensemble averaging)
  - examples of applications
Multivariate autoregressive model (MVAR) fitted to the \( k \)-channel EEG process is expressed as:

\[
X(t) = \sum_{i=1}^{p} A(i)X(t-i) + E(t)
\]

Where: \( X \) - vector of \( k \) signals recorded in time: \( X(t) = (X_1(t), X_2(t), ..., X_k(t)) \), \( E(t) \) is the vector of white noise values, \( A(i) \) are the model coefficients and \( p \) is the model order.

Model order \( p \) is found from criteria developed on the grounds of information theory, the most popular is Akaike (AIC) criterion:

\[
AIC(p) = 2 \log(\det(V)) + 2kp / N
\]

\( V \)-matrix of noise variance
AR model coefficients are found by minimalisation of the variance matrix. This leads to the calculation of the correlation matrix.

The elements of correlation matrix $R(s)$ are found as:

$$R_{ij}(s) = \frac{1}{N_S} \sum_{t=1}^{N_S} X_i(t)X_j(t+s), \quad \text{for } s = 0, \ldots, p$$

Depending on the version of the estimators used, we may have $N_S = n - |s|$ for unbiased correlation estimator and $N_S = n$ for biased estimator ($n$ is the record length). The later ensures a positive definite form of $R(s)$. The AR model parameters may be obtained by solving the set of linear equations (called Yule-Walker equations)

$$
\begin{pmatrix}
R(0) & R(-1) & \cdots & R(p-1) \\
R(1) & R(0) & \cdots & R(p-2) \\
\vdots & \vdots & \ddots & \vdots \\
R(1-p) & R(2-p) & \cdots & R(0)
\end{pmatrix}
\begin{pmatrix}
A(1) \\
A(2) \\
\vdots \\
A(p)
\end{pmatrix}
=
\begin{pmatrix}
R(-1) \\
R(-2) \\
\vdots \\
R(-p)
\end{pmatrix}
$$

$$V = \sum_{j=0}^{p} A(j)R(j) \quad V \text{ - variance matrix of a noise}$$
Assuming \( A(0) = -I \) (\( I \) - identity matrix), we can rewrite the autoregressive dependence in another form:

\[
\mathbf{E}(t) = \sum_{j=0}^{p} A(j) \mathbf{X}(t - j)
\]

Transforming the above equation to the frequency domain by application of \( Z \) transform we get:

\[
\mathbf{X}(f) = A^{-1}(f) \mathbf{E}(f) = \mathbf{H}(f) \mathbf{E}(f)
\]

\[
z = e^{-2\pi f \Delta t}
\]

The \( \mathbf{H}(f) \) is called the transfer matrix of the system. \( f \) denotes frequency,
MVAR model in the frequency domain

Transfer matrix $H(f)$ defined in the frequency domain contains spectral information and phase dependencies between channels.

From $H(f)$ spectra and coherences can be found
From transfer matrix $H(f)$ spectrum and coherences may be found

**Spectrum**

$$S(f) = H(f) \mathbf{V} H^*(f)$$

$V$ is a variance matrix of a noise

**Ordinary coherence:**

$$k_{ij}^2(f) = \frac{S_{ij}^2(f)}{S_{ii}(f)S_{jj}(f)}$$

**Partial coherence:**

$$\chi_{ij}^2(f) = \frac{M_{ij}^2(f)}{M_{jj}(f)M_{ii}(f)}$$

**Multiple coherence:**

$$\mu_j(f) = (1 - \det|S(f)|/S_{jj}(f)M_{jj}(f))^{1/2}$$

$M_{ij}$ is a minor of spectral matrix
The multivariate approach allows for easy calculation of spectra, ordinary, partial and multiple coherencies.

\[ S(f) = H(f) V H^*(f) \]

Partial coherence:

\[ \chi_{ij}^2(f) = \frac{M_{ij}^2(f)}{M_{jj}(f)M_{ii}(f)} \]

Multiple coherence:

\[ \mu_j(f) = (1-\text{det}|S(f)|/S_{jj}(f)M_{jj}(f))^{1/2} \]

\( M_{ij} \) is a minor of spectral matrix.
GRANGER CAUSALITY\(^1\) is based on the predictability of the series:

If we try to predict a value of \(X(t)\) using \(p\) previous values of the series \(X\) only, we get a prediction error \(e_1\):

\[
X(t) = \sum_{j=1}^{p} A_{11}(j)X(t - j) + e_1(t)
\]

If we try to predict a value of \(X(t)\) using \(p\) previous values of the series \(X\) and \(q\) previous values of \(Y\) we get another prediction error \(e_2\):

\[
X(t) = \sum_{j=1}^{p} A_{11}(j)X(t - j) + \sum_{j=1}^{p} A_{12}(j)Y(t - j) + e_2(t)
\]

If the variance of \(e_2\) (after including series \(Y\) to the prediction) is lower than the variance of \(e_1\), we say that \(Y\) causes \(X\) in the sense of Granger causality.

*If some series \(X_1(t)\) contains information in past terms that helps in the prediction of series \(X_2(t)\), then \(X_1(t)\) is said to cause \(X_2(t)\) - this formulation is compatible with the MVAR model*

The concept of GRANGER CAUSALITY was originally defined for two signals, however as was pointed out by Granger (1980) in his next publication test of causality is impossible unless the set of interacting channels is complete.

What, if we have more than 2 interacting channels?

This is usually the case for EEG signals.

The concept of Granger causality was extended for arbitrary number of channels by introduction of Directed Transfer Function\(^1\) (DTF) and later Partial Directed Coherence\(^2\).

Both measures are based on Multivariate Autoregressive Model.

DTF


PDC

DTF – Directed Transfer Function

In the transfer matrix $H(f)$ not only frequency information but also phase dependencies between channels are coded. The future of one channel can be predicted from the past of other channels.

The DTF function, describing transmission from channel $j$ to $i$ at frequency $f$ normalized in respect of inflows to channel $i$ is defined as:

\[
\gamma_{ij}^2(f) = \frac{|H_{ij}(f)|^2}{\sum_{m=1}^{k}|H_{im}(f)|^2}
\]

Non normalized DTF

\[
\theta_{ij}^2(f) = |H_{ij}(f)|^2
\]

Non-normalised DTF for two signals is equivalent to Granger causality, it is proportional to the coupling strength between signals.

LFPs were simulated by a model composed of N coupled cortical columns where each column was made up of an excitatory and inhibitory neuronal population (Freeman 1992).

M. Kamiński, M. Ding, W. Truccolo, S.L. Bressler,

*Evaluating causal relations in neural systems: Granger causality, directed transfer function and statistical assessment of significance.*

DTF detects correctly one way and reciprocal connections in point processes and continuous time series.

DTF is robust in respect to random noise

\[
\text{EEG} + \text{noise} = \text{signal in 1st chan.}
\]

EEG – experimental EEG from electrode P3, relaxed state

Δ – delay

noise – random noise from Gaussian distribution

noise amplitude/EEG amplitude = 3
The partial directed coherence was defined by Baccala and Sameshima in the following form:

\[ P_{ij}(f) = \frac{A_{ij}(f)}{\sqrt{a_j^*(f)a_j(f)}} \]

In the above equation \( A_{ij}(f) \) is an element of \( A(f) \) – a Fourier transform of model coefficients \( A(t) \), \( a_j(f) \) is \( j \)-th column of \( A(f) \) and the asterisk denotes the transpose and complex conjugate operation. Although PDC is a function operating in the frequency domain, the dependence of \( A(f) \) on the frequency has not a direct correspondence to the power spectrum. PDC is a multivariate method which detects direct connections.

Bivariate coherence

Simulation

Result
Bivariate Granger Causality

Simulation

Result
Non-normalized DTF

Simulation

Result

\[
\begin{align*}
\Delta &= 1 \\
\Delta &= 2 \\
\Delta &= 3 \\
\Delta &= 4
\end{align*}
\]
The "leak currents" indicating the accuracy of DTF are found from surrogate data - signals with disturbed phases. Surrogate data are obtained by transforming the data to the frequency domain, randomizing their phases and transforming back to the time domain.
Simulation scheme

Sampling rate = 128Hz

Length of signal = 10s

Number of points per channel = 1280
Granger causality (pair-wise)

simulation:

result:
Directed Transfer Function (DTF)

simulation:

result:
In some cases such as e.g. recording from depth electrodes, where many coupled structures are communicating along many possible pathways, the identification of direct activity flows is important. To resolve this problem direct Directed Transfer function was proposed. It is obtained by multiplication of DTF by partial coherence:

$$\delta_{ij}(f) = \chi_{ij}(f) \eta_{ij}(f)$$

$$\text{dDTF}(f) = (\text{partial coherence } (f)) \ (\text{DTF}(f))$$

**Direct Directed Transfer Function (dDTF)**

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**Simulation:**

![Simulation Diagram](image50.png)

**Result:**

![Result Diagram](image51.png)
Application of DTF and DDTF to local field potentials

Animal walking on a runway, stressing stimulus - bell
Signals registered by electrodes chronically implanted in brain structures:

BLA- basolateral amygdala
VSB- ventral subiculum
ACC- nucleus accumbens
SPI- subpallidal area
MVAR model was fitted to 21 channels of sleep EEG (10/20 system) simultaneously and DTF functions were determined.
EEG activity propagation during different sleep stages
(averaged over 9 subjects)
Comparison of different methods of directionality estimation for EEG (awake eyes open)
DTF and PDC

PDC detects only direct connections, DTF direct and indirect connections (in order to identify only direct connections DDTF has to be used). Although PDC is a function operating in the frequency domain, the dependence of $A_{ij}(f)$ on the frequency has not a direct correspondence to the power spectrum. Other differences are connected with normalisation. In DTF the outflow from channel $j$ to $i$ are normalised by all inflows to channel $i$.

In case of PDC the inflow to channel $i$ described by the element $A_{ij}(f)$ is divided by the outflows from $j$ to all other channels. In consequence even weak sources emitting activity in few directions are emphasised.

PDC shows rather sinks than sources, DTF shows sources.
The input data for the MVAR model should not be subjected to pre-processing, which introduces correlation between signals.

No „common average”, „source derivation”, Hjorth or Laplace transform may be used, since they disturb the correlations and phases between channels.

Directed Transfer Function is insensitive in respect to volume conduction, since DTF detects phase differences and volume conduction is zero phase propagation. DTF is robust in respect of random phase noise and constant phase disturbances.
DTF and PDC are multivariate methods based on MVAR model. When fitting models to the data the number of coefficients should be much smaller than the number of data points (preferably <0.1).

DTF statistical properties depend on the ratio of:

\[
\frac{\text{number of MVAR coefficients}}{\text{number of data points}} = \frac{k^2p}{kN} = \frac{kp}{N}
\]

Where \( k \) – number of channels, \( p \) – model order, \( N \) – number of data points

Number of channels is limited by the length of the stationary data window.

This difficulty was overcome by introduction of SDTF.

**Time varying - Short time Directed Transfer Function (SDTF) can be found when multiple realisations are available.**
When multiple realizations are available the model coefficients $a_i$ can be estimated by an ensemble averaging of correlation matrix:

$$\tilde{R}_{ij}(s) = \frac{1}{N_T} \sum_{r=1}^{N_T} R_{ij}^{(r)}(s) = \frac{1}{N_T} \sum_{r=1}^{N_T} \frac{1}{n-s} \sum_{t=1}^{n-s} X_i^{(r)}(t) X_j^{(r)}(t-s)$$

Then MVAR model can be fitted to short data epochs and Short-time Directed Transfer Function (SDTF) can be determined.
Time course of the propagation of beta activity after movement
Experiment – finger movement paradigm

- Sound
- Cue
- Analyzed epoch
Beta activity propagation during finger movement and its imagination
Gamma activity propagation during finger movement and its imagination
Gamma band, fist clenching
DTF can be used to evaluate causality between point processes e.g. spike trains. Spikes are low-pass filtered (Butterworth filter) and 10% noise is added.

Rat hippocampal LFP were measured together with firing of 12 SUM (Supramammillary) neurons. Neuronal population was inhomogenous – some neurons fired on positive peak of CA1 theta waves, others on their raising phase. The directions of propagation were evaluated by means of SDTF for spontaneous activity and during stimulus – tail pinch.

B. Kocsis, M. Kaminski *Dynamic changes in the direction of the theta rhythmic drive between supramammillary nucleus and the septohippocampal system*; Hippocampus 16(6):531-540, 2006 (available online).
Results

During rest DTF showed predominant hippocampus-to-SUM direction of influence. In the episode of sensory stimulation the direction of drive from SUM to hippocampus was observed.

The study conducted by means of SDTF revealed dynamic relationship between SUM and septohippocampal theta oscillations in which the direction of the rhythmic drive changes depending on the origin and type of the rhythm.
Directed Transfer Function - DTF

• 1991 Introduction of Directed Transfer Function

• Localization of epileptic foci

• Propagation in brain structures during locomotion
  A. Korzeniewska, S. Kasicki, M. Kaminski, K. J. Blinowska. Information flow between hippocampus and related structures during various types of rat’s behavior. Journal of Neuroscience Methods, 73, pp. 49-60, 1997

• Topography of EEG propagation during sleep

Short-time Directed Transfer Function SDTF


• B.Koscis, M. Kaminski, Dynamic changes in the direction of the theta rhythmic drive, Hippocampus, 2006, 16:531-540.
For mutually dependent set of signals the correct pattern of propagations can be found only if all relevant channels are evaluated simultaneously.

DTF is an estimator robust in respect to noise, proportional to coupling between channels and capable to identify reciprocal connections.

When multiple realizations of experiment are available SDTF can be estimated and the propagation as a function of time and frequency can be found.
MP and DTF methods are complementary

- MP has higher resolution in time and frequency, makes possible identification and parametrisation of transients and rhythmical activity in the framework of the same formalism.
- DTF is a multivariate method, it gives the information not only about amplitude, but also about phase dependencies between signals and hence on their propagation.
- The information about both methods, references, papers and some software can be found at portal EEG.PL.
Welcome to EEG.pl

EEG.pl is an open repository for software, publications and datasets related to the analysis of brain potentials: electroencephalogram (EEG), local field potentials (LFPs) and event related potentials (ERP), created to foster and facilitate Reproducible Research in these fields.

You can freely search the content of this and other thematic portals linked via the inter-neuro initiative. As a registered user you can submit your article, data or model. Registration and submissions are free. You can also comment and respond to comments on any of the published items.

We invite you to check SignalML - the solution to the problem of incompatibility of EEG formats, software for the matching pursuit decomposition of signals and demonstration of the the Directed Transfer Function.

Service by Laboratory of Medical Physics, Warsaw University

Supported by grant 4439/14/115/2003 from the Polish State Committee for Scientific Research.

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DTF

Determination of EEG propagation in brain

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**Introduction**

The determination of the biological signals propagation and in particular directions of flow of the brain activity are crucial for the understanding of information processing in organisms. Several methods have been proposed for estimation of the directionality, however they were defined for two channels only. When larger set of interacting channels is involved bivariate estimates lead to erroneous results. DTF -- Directed Transfer Function (Kamiński and Blinowska 1991 [1]) takes into account all channels of process simultaneously and makes possible estimation of activity flow in given direction as a function of frequency. It is based on the concept of Granger causality [13] extended for arbitrary number of channels (Kamiński et al. 2001 [3]). DTF is robust in respect to noise and constant phase disturbances, in particular it discriminates against volume conduction, which propagates with zero phase. Short-time Directed Transfer Function (SDTF) makes possible calculation of activity flow as a function of time and frequency, when multiple realizations of the processes are available (e.g. event related responses).
MP interactive calculations online

"Load/Build" button refers to the signal to be MP-decomposed.
THANK YOU FOR YOUR ATTENTION!